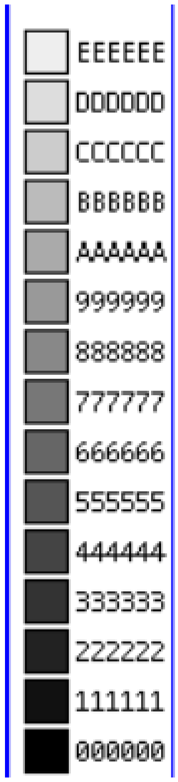
Contents

The Hexadecimal Numbering System



One common way of overcoming the one main disadvantage of binary numbers where the binary string equivalent of a large decimal base-10 number can be quite long is to arrange the binary numbers into groups or sets of four bits (4-bits).

Hexadecimal numbers, Hex, are "natural" to computers, even though computers don't actually work in Hex. This is because computers store and handle binary digits, and a group of four binary digits make one hex digit.

BASE 10	BASE 2	BASE 16	BASE 8
DECIMAL	**BINARY**	**HEXADECIMAL**	**OCTAL**
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4

Bytes are typically 8 bits, and can store the values 0 – 255 ($0000\ 0000_2$ to $1111\ 1111_2$)

Since a byte is 8 bits, it makes sense to divide that up into two groups, the top 4 bits and the low 4 bits.

4-bit Binary	**Decimal Equivalent**
0000_2	0_{10}
1111_2	15_{10}

Since 4 bits gives you the possible range from 0 – 15, a base 16 system is easier to work with

i.e. It's easier to express for us to express a value as "A" then it is to express it as "1010"

Hex and computing

[ASIDE: In computer architecture, *8-bit* integers, memory addresses, or other data units that are *8 bits* wide are called 1 *octet* wide - the eight binary digits in a byte has been called *an octet*]

Octal was originally used by computer programmers because all opcodes (a single executable machine language instruction) for the Intel 8080 (the engine of the Altair 8800) which was released in 1974, were 8-bit (8-bit = 1 octets).

Now think about these programs for the Inter 8080… with a hexadecimal (base 16) representation, you could enter twice as much information with each keystroke compared to octal (base 8). As a result, things tended to be expressed in hex.

Hex and colours

Users need a way to describe three different intensity (about Red, Green and Blue) with a number that is wide enough to grant enough levels so that our eyes cannot distinguish.

Computer processors are more efficient if the number of values is a power of two (so that full use is made of the bits combinations) and 256 values are a good match to our eyes' perception of colours:

> 8-bit colour graphics is a method of storing image information in a computer's memory or in an image file, such that each pixel is represented by one 8-bit byte. The maximum number of colours that can be displayed at any one time is 256

In the early days of computing, many displays were only capable of displaying 256 colours

Each hexadecimal digit represents four binary digits, also known as a nibble, which is half a byte.

For example, one single byte can have values ranging:

$0000\ 0000_2$ to **$1111\ 1111_2$** or more conveniently represented as **00_{16}** to **FF_{16}**

The RGB values were specified as a tuple of 3 octets, which is much shorter in hexadecimal (where 1 character can represent all 8 bits) than in octal (where 3 values represent 9 bits, with 1 bit not needed).

Later standards called for more colours, and thus it became easier to specify those colours in hexadecimal and were best represented in hexadecimal for readability and space savings

One byte represents a number in the range 00 to FF (in hexadecimal notation), or 0 to 255 in decimal notation.

This represents the least (0) to the most (255) intensity of each of the colour components. Thus web colours specify colours in the True Colour (24-bit RGB) colour scheme.

The hex triplet is formed by concatenating three bytes in hexadecimal notation, in the following order:

Byte 1: red value (colour type red)

Byte 2: green value (colour type green)

Byte 3: blue value (colour type blue)

16 Million Colours:

Because each of the three colours can have values from 0 to 255 (256 possible values), there are:

$256 \times 256 \times 256$ $= 256^3$ possible colour combinations

$= 16{,}777{,}216$ possible colour combinations

and this is why you see claims of "16 Million Colours" on computer equipment

For example, here is an RGB pallet represented:

in hex:	in octal:
F3 C7 B2	747436662

Currently, most graphics hardware runs in 24-bit truecolour or 32-bit truecolour

Colour references in HTML and CSS can be expressed with six hexadecimal digits (two each for the red, green and blue components, in that order) prefixed with #: white, for example, is represented #FFFFFF.

In the RGB model, we regulate each colour from 0 (no light) to 255 (full saturation). Using HEX, that the top value for colour (256) is represented by FF (2 digits), which makes it convenient and easy for us to understand where white in decimal would be 255255255_{10}, while in hex is simply $FFFFFF_{16}$

CSS allows 3-hexdigit abbreviations with one hexdigit per component: #FA3 abbreviates #FFAA33

`#AABBCC` makes it clear to know what the three values are (AA,BB and CC), but `11189196` (the equivalent decimal) does not: what is Red, Green and Blue?

Convert from binary to hexadecimal

To convert from binary to hexadecimal is to ***group*** binary digits into sets of **four**, starting with the least significant (rightmost) digits.

Binary: 11100101_2 = 1110 0101_2

Then, look up each group in a table:

Binary:	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
Hexadecimal:	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F

For example:

Binary =	1110	0101	
Hexadecimal =	E	5	= E5 hex

So, $\mathbf{11100101_2 = E5_{16}}$

REMEMBER: Group the binary string into groups of 4, starting on the right from (LSB) to the left (MSB).

4-bits are also required to produce a hexadecimal number, so a hex digit can also be thought of as half-a-byte.
Therefore, two hexadecimal numbers are required to produce one full byte ranging from 00_{16} to FF_{16} (which is equal to decimal 255_{10})

→ The maximum 4-digit hexadecimal number is $FFFF_{16}$ which is equal to $65{,}535_{10}$ and so on

→ four digits in a binary number can be represented with a single hexadecimal digit

→ hexadecimal can be used to write large binary numbers with much fewer digits

→ The numbers 0 to 9 are still used as in the original decimal system, but the numbers from 10 to 15 are now represented by capital letters of the alphabet from A to F inclusive

11101010_2			
Group the bits into four's starting from the right hand side			
=	1110	1010	
Find the Decimal equivalent of each individual group			
=	14	10	(in decimal)
Convert to Hexadecimal using the table above			
=	E	A	(in Hex)
Then, the hexadecimal equivalent of the binary number $1110\ 1010_2$ is #EA_{16}			

Convert the following from Binary to Hex:

a) 100101111010_2
b) 1111110011_2
c) 101010101010_2
d) 000011001111_2

(a) **1001 0111 1010**

Looking at the table above, you can now calculate the **Decimal** equivalent:

1001 = 9 0111 = 7 1010 = 10

9 **7** 10

10 represented in Hex = **A**

So, $100101111010_2 = 97A_{16}$

(b) **11 1111 0011**

Remembering that if you have less than 4 bits in a binary string, then you must pre-pin the binary string with 0's to make the string complete. These become the (most significant bit) MSB on that number but do not change the number itself or its result.

0011 1111 0011

Now working converting them to **Decimal (results)**:
0011 = 3 1111 = 15 0011 =3

0011 = 3 1111 = 15 0011 = 3

Hex: 3 F 3

$1111110011_2 = 3F3_{16}$

It's much easier to convert from to other formats once you have converted to Binary.

EXERCISE: Complete for (c) and (d) above

Convert from decimal to hexadecimal

The most convenient way to do this is to convert the decimal number to binary and then, as above, group the binary into groups of 4 from the right/LSB and determine the Hex representation for that group of 4 binary bits

<8mins video desc of hex numbering system representation from 2012 where he mentions "top end 64-bit" computer ☺ https://www.youtube.com/watch?v=9xbJ3enqLnA

There are 16 types of people in the world
Those who understand hexadecimal and F the rest

Number Bases Reference Table

BASE $_{10}$	BASE $_{2}$	BASE $_{16}$	BASE $_{8}$
DECIMAL	**BINARY**	**HEXADECIMAL**	**OCTAL**
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5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	8	10
9	1001	9	11
10	1010	A	12
11	1011	B	13
12	1100	C	14
13	1101	D	15
14	1110	E	16
15	1111	F	17